



Máquinas Eléctricas

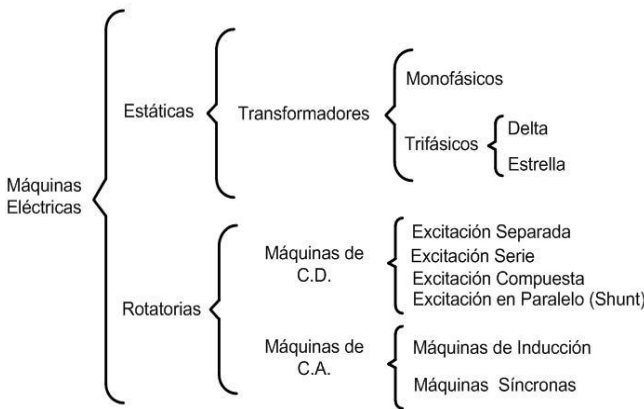


Contenido

- 1 Máquinas Eléctricas
- 2 Circuitos Magnéticos
- 3 Transformador
 - Transformador ideal
 - Transformador real
 - Transformador Trifásico
- 4 Conversión de energía electromecánica
 - Ley de Fuerza de Lorentz
 - Método de Energía y Coenergía
- 5 Máquina de CD
- 6 Máquina de Inducción
- 7 Máquina Síncrona



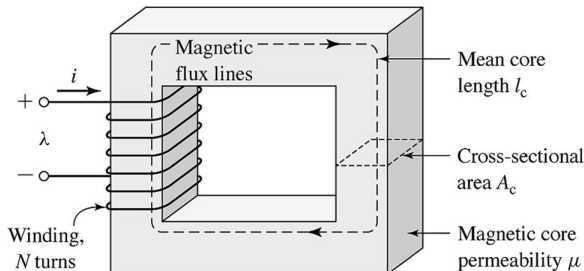
Máquinas Eléctricas





Circuito Magnético

$$\oint_C \underbrace{\mathbf{H}}_{[A \cdot \text{Vueltas}]} \cdot d\mathbf{l} = \int_S \underbrace{\mathbf{J}}_{[A/m^2]} \cdot d\mathbf{a}; \quad \oint_S \underbrace{\mathbf{B}}_{[Wb/m^2=1T]} \cdot d\mathbf{a} = 0$$



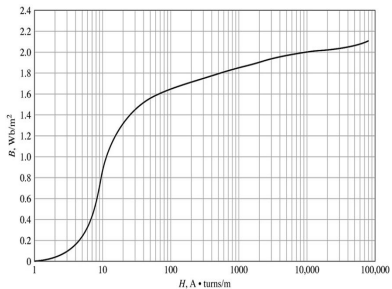
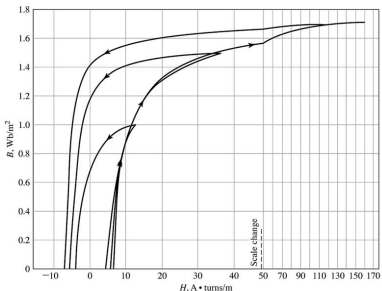
$$\underbrace{\phi_c}_{[Wb]} = \int_S \mathbf{B} \cdot d\mathbf{a} = B_c A_c; \quad \underbrace{\mathcal{F}}_{[A \cdot \text{Vueltas}]} = Ni = \oint \mathbf{H} \cdot d\mathbf{l} = H_c l_c$$



Circuito Magnético (Acero Eléctrico M-5-0.012)

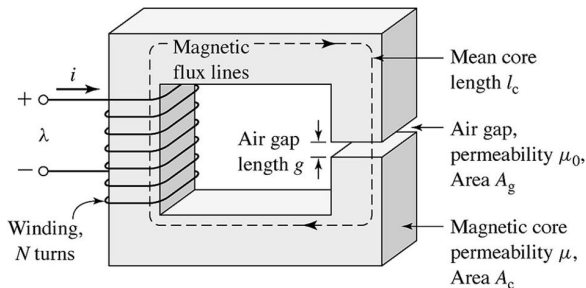
$$\mathbf{B} = \mu \mathbf{H}$$

- 1 μ es la permeabilidad magnética del material [H/m]
- 2 $\mu = \mu_r \mu_0$; $\mu_0 = 4\pi \times 10^{-7}$ [H/m] (2000, 80000)





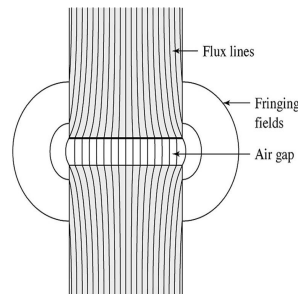
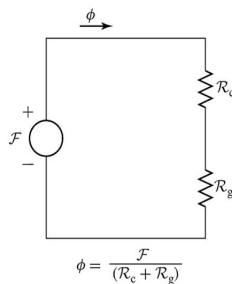
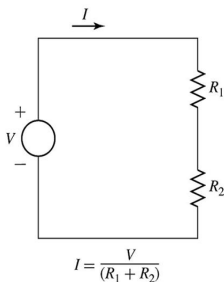
Circuito Magnético-Entrehierro (GAP)



$$B_c = \frac{\phi}{A_c}; B_g = \frac{\phi}{A_g}$$

$$\mathfrak{S} = H_c l_c + H_g g = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g = \phi \left(\underbrace{\frac{l_c}{\mu A_c}}_{\mathfrak{R}_c} + \underbrace{\frac{g}{\mu A_g}}_{\mathfrak{R}_g} \right)$$

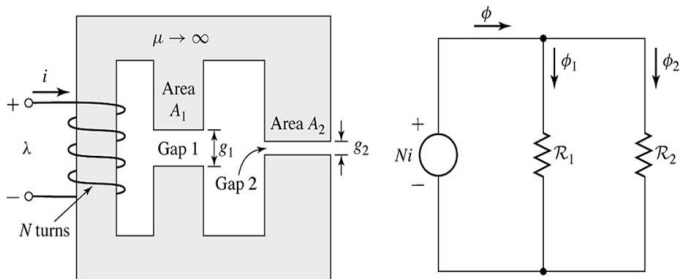
Circuito Eléctrico-Magnético



$$\phi \approx \frac{\mathfrak{S}}{\mathcal{R}_g} = \frac{\mathfrak{S} \mu_0 A_g}{g} = Ni \frac{\mu_0 A_g}{g}$$



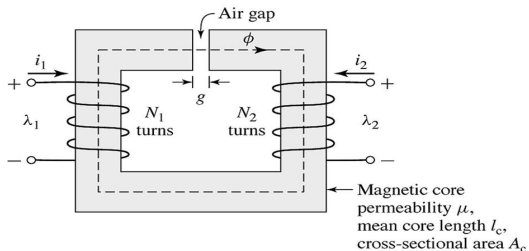
Circuito Eléctrico-Magnético: Ejemplo 1



$$L = \frac{\lambda}{i} = \frac{N\phi}{i}$$



Circuito Eléctrico-Magnético: Ejemplo 2



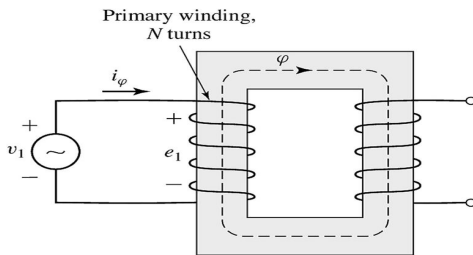
$$\mathfrak{F} = N_1 i_1 + N_2 i_2 \Rightarrow L = (N_1 i_1 + N_2 i_2) \frac{\mu_0 A_c}{g}$$

$$\lambda_1 = N_1 \phi = N_1^2 \left(\frac{\mu_0 A_c}{g} \right) i_1 + N_1 N_2 \left(\frac{\mu_0 A_c}{g} \right) i_2 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = N_2 \phi = N_1 N_2 \left(\frac{\mu_0 A_c}{g} \right) i_1 + N_2^2 \left(\frac{\mu_0 A_c}{g} \right) i_2 = L_{21} i_1 + L_{22} i_2$$



Transformador monofásico en vacío



$$e = \frac{d\lambda}{dt} = N_1 \frac{d\phi}{dt}$$

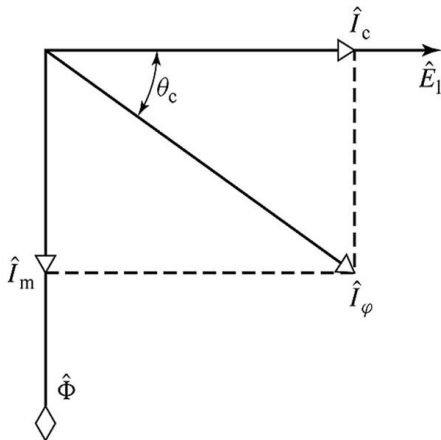
$$u_1 = R_1 i_\phi + e_1$$

$$\phi = \phi_{max} \sin \omega t \Rightarrow e = N_1 \frac{d\phi}{dt} = \omega N_1 \phi_{max} \cos \omega t$$

$$\phi_{max} = \frac{V_1}{\sqrt{2} \pi f N_1}$$



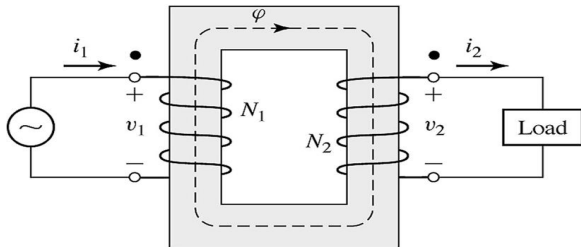
Diagrama Fasorial



$$P_c = E_1 I_\phi \cos \theta_c$$



Transformador monofásico con carga



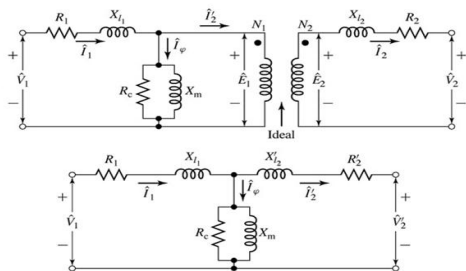
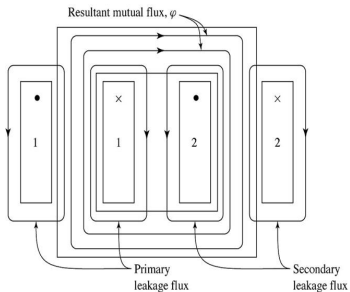
$$u_1 = e_1 = N_1 \frac{d\phi}{dt}; u_2 = e_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{u_1}{u_2} = \frac{N_1}{N_2}; \frac{i_2}{i_1} = \frac{N_2}{N_1}$$

$$u_1 i_1 = u_2 i_2$$

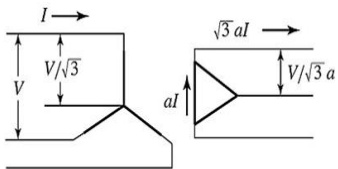


Circuito Equivalente

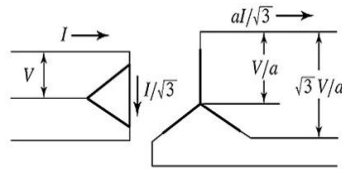


$$u_1 i_1 \approx u_2 i_2$$

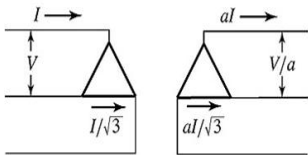
Transformador monofásico con carga



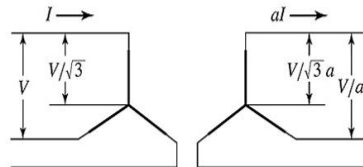
Y-Δ connection



Δ-Y connection



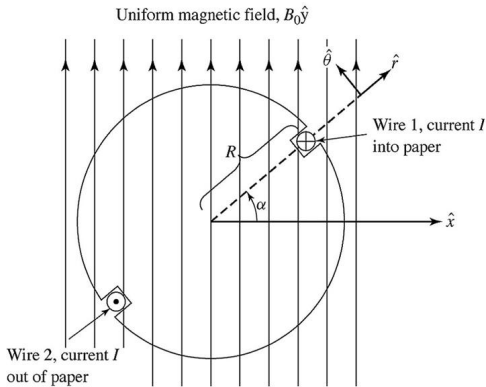
Δ-Δ connection



Y-Y connection



Ley de Fuerza de Lorentz



$$\mathbf{F} = \mathbf{I} \times \mathbf{B}$$



Balance de Energía

Energía de entrada de fuentes eléctricas = Energía mecánica de salida + Incremento en la energía almacenada en el campo magnético + Energía convertida en calor

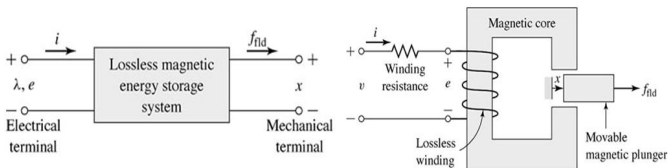
$$\frac{dW_{fld}}{dt} = \underbrace{e}_{\frac{d\lambda}{dt}} i - f_{fld} \frac{dx}{dt}$$

$$dW_{elec} = (ie)dt = dW_{Mech} + dW_{fld}$$

- 1 dW_{elec} = diferencial de la energía eléctrica de entrada
- 2 dW_{Mech} = diferencial de la energía mecánica de salida
- 3 dW_{fld} = diferencial de la variación en la energía magnética almacenada



Método de Energía

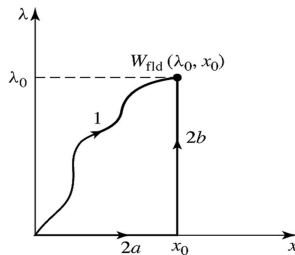


$$\frac{dW_{fld}(\lambda, x)}{dt} = \underbrace{e}_{\frac{d\lambda}{dt}} i - f_{fld} \frac{dx}{dt}$$

$$dW_{fld}(\lambda, x) = id\lambda - f_{fld}dx = \underbrace{\frac{\partial W_{fld}(\lambda, x)}{\partial \lambda}}_i d\lambda + \underbrace{\frac{\partial W_{fld}(\lambda, x)}{\partial x}}_{-f_{fld}} dx$$



Método de Energía

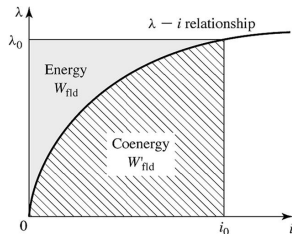


$$W_{fld}(\lambda_0, x_0) = \int_{Tray2a} dW_{fld} + \int_{Tray2b} dW_{fld}$$

$$W_{fld}(\lambda_0, x_0) = \int_0^{\lambda_0} i(\lambda', x_0) d\lambda' = \int_0^{\lambda_0} \frac{\lambda'}{L(x)} d\lambda' = \frac{1}{2} \frac{\lambda^2}{L(x)}$$



Coenergía

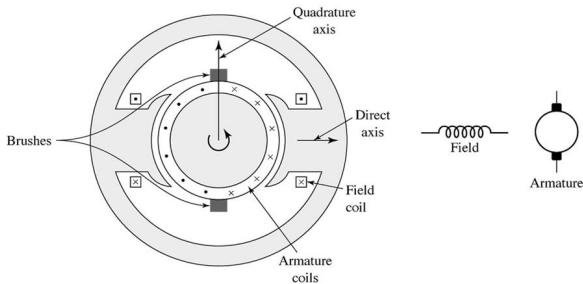


$$W'_{fld}(i, x) = i\lambda - W_{fld}(\lambda, x)$$

$$dW'_{fld}(i, x) = \lambda di + f_{fld} dx = \underbrace{\frac{\partial W'_{fld}(i, x)}{\partial i}}_{\lambda} di + \underbrace{\frac{\partial W'_{fld}(i, x)}{\partial x}}_{f_{fld}} dx$$



Máquina de CD



$$e_a = \left(\frac{\text{Polos} C_a}{2\pi m} \right) \phi_d \omega_m = K_a \phi \omega_m$$

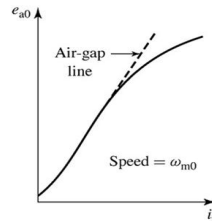
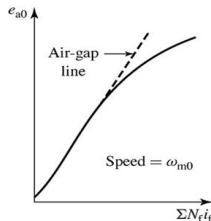
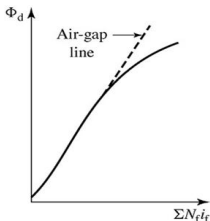
$$e_a i_a = T_{Mech} \omega_m$$

$$T_{Mech} = K_a \phi_d i_a$$



Máquina de CD

$$\phi_d = \frac{1}{\mathcal{R}} \sum N_f i_f$$

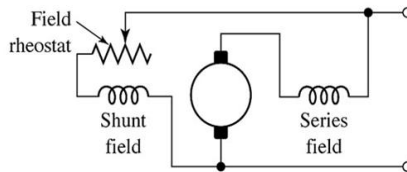
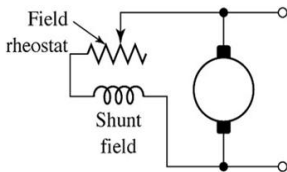
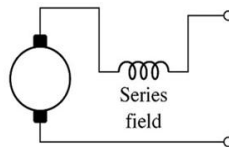
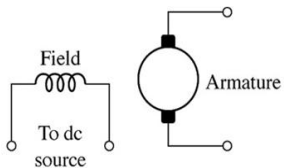


$$\frac{e_a}{\omega_m} = K_a \phi_d = \frac{e_{a0}}{\omega_{m0}}$$

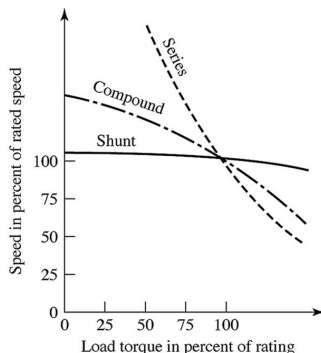
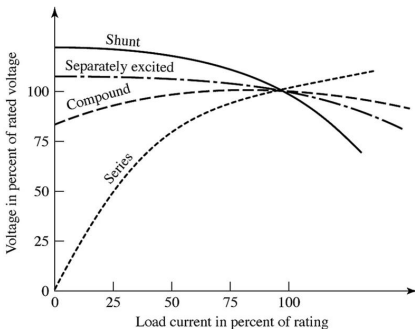
$$e_a = \frac{\omega_m}{\omega_{m0}} e_{a0} = \frac{n}{n_0} e_{a0}$$



Máquinas de CD

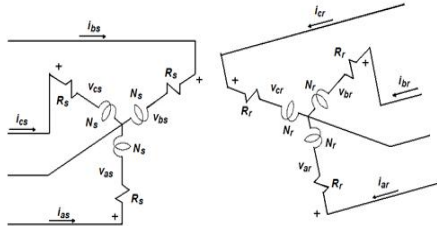
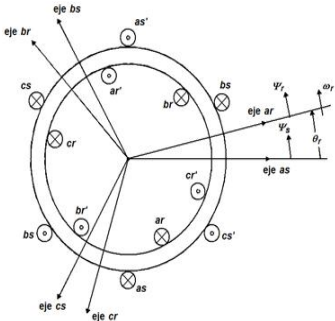


Características Generador-Motor CD



$$V_a = E_a \pm I_a R_a$$

Máquina de Inducción





Máquina de Inducción

Principio de Funcionamiento de la máquina de inducción

El campo magnético induce un fuerza electromotriz en el rotor

$$N_r = \frac{120 f}{P}$$

Circulan corrientes por el rotor

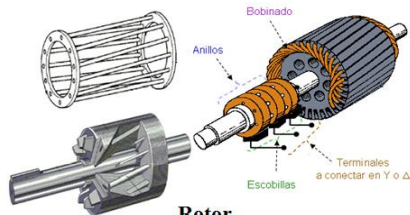
Fuerzas electromagnéticas entre los campos magnéticos del rotor y el estator

Par en el rotor: el rotor gira

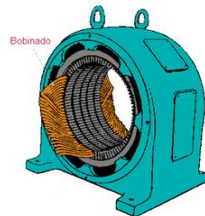
El rotor gira a una velocidad N_r inferior a la velocidad de sincronismo N_s . Velocidad relativa conocida como deslizamiento (S)

$$S = \frac{N_s - N_r}{N_s}$$

En caso contrario no se inducirán fuerzas electromotrices en el rotor y el por lo tanto, no habría par motor

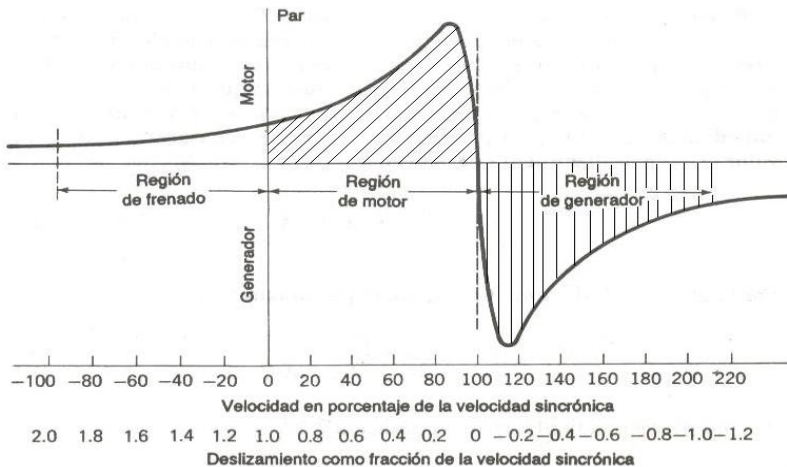


Rotor



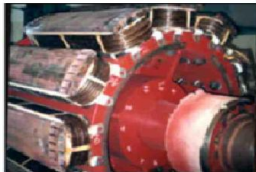


Máquina de Inducción

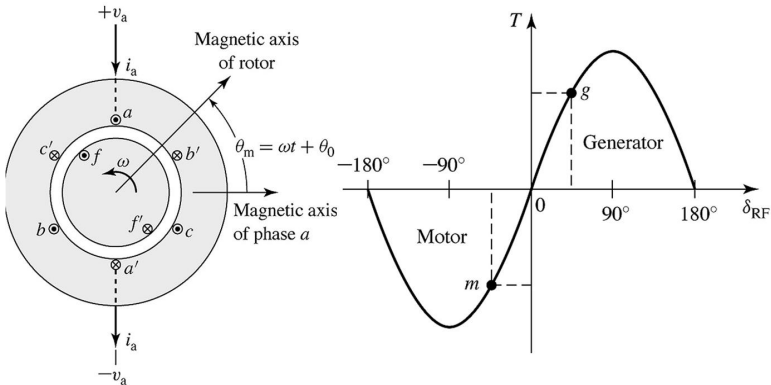




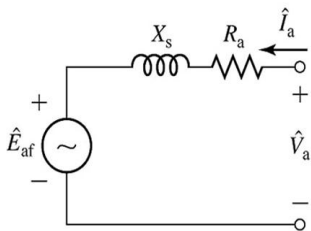
Máquina Síncrona



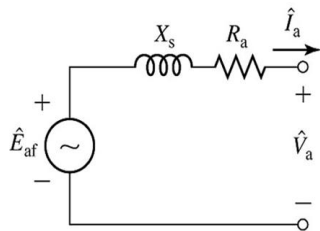
Máquina Síncrona



Máquina Síncrona



Motor

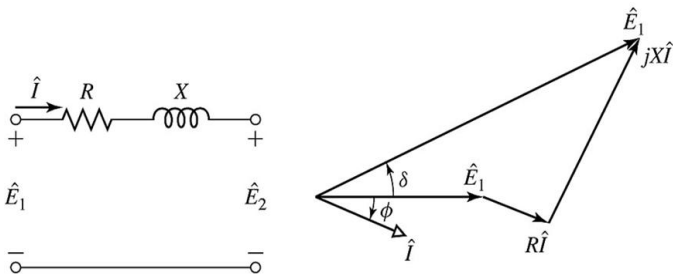


Generador

$$\tilde{V}_a = \pm R_a \tilde{I}_a \pm jX_s \tilde{I}_a + \tilde{E}_{af}$$

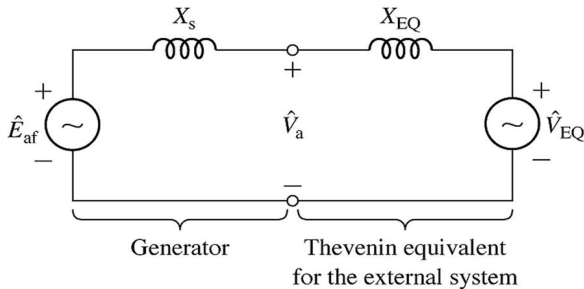


Máquina Síncrona



$$P_1 = P_2 = \frac{E_1 E_2}{X} \sin \delta$$

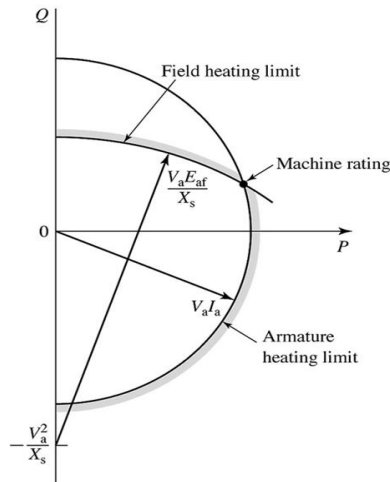
Máquina Síncrona



$$P = \frac{E_{af} V_{EQ}}{X + X_{EQ}} \sin \delta$$

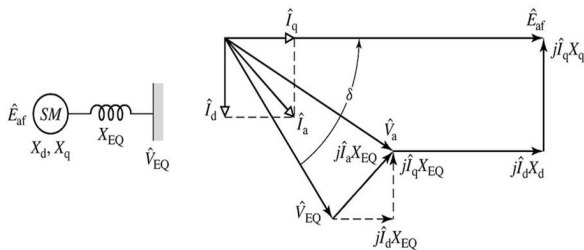


Máquina Síncrona





Máquina Síncrona



$$P = \frac{E_{af} V_{EQ}}{X_{dt}} \sin \delta + \frac{V_{EQ}^2 (X_{dT} + X_{qT})}{2X_{dT} X_{qT}} \sin 2\delta$$

Máquina Síncrona

