Transient Analysis of a Synchronous Generator Using a High-Order State Space Representation

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Abstract

A state-space representation of a Synchronous generator is established and used to analyze the transient behavior of a Synchronous Generator. The machine is a 2 pole, 150 MVA, 120 MW, 13.8 kV, 50 Hz generator connected to an infinite bus through a transformer and a transmission line. The state-space representation is based on a two-axis equivalent-circuit model of the generator and includes the swing equation and the voltage regulator. The two-axis representation of the infinite bus voltage is chosen as reference. A three phase short circuit is simulated at the transformer terminals. Graphic results are analyzed and compared with actual measurements and with those obtained from a finite element model of the generator. The results give insight into the electromagnetic phenomena and the state-space representation provides a platform for the analysis of control schemes to improve the stability of the generator.

1. Introduction

The transient behavior of synchronous machines is of great interest to power system engineers. Stability of a power network depends mainly on the response of generators to important disturbances. Hence the phenomena present after fault occurrence is an important study subject. In this paper the transient analysis is performed on a Synchronous Generator (SG) connected to a system that can be represented as an infinite bus. The approach is to establish a state variable representation of the system and to find its response to a three phase short circuit. The intention is to validate the state-space (SS) representation, which will be used to analyze new control approaches to improve the SG response.

A state variable model gives a complete picture of a system, as it includes a description of the internal status of the system as well as the input-output behavior, thus guaranteeing strong couplings among all the systems included [1]. In this paper, the chosen state variables for the analysis of the electromechanical system (turbo generator connected to the electrical system) are currents of all windings, angular velocity, load angle and field voltage. It is important to mention that any number of damper branches can be simulated and that transients associated with the transmission network are considered.

2. System description and two-axis representation

The SS model proposed here is used to analyze the behavior of a SG connected to an infinite bus through a transformer and a transmission line, when a three phase short circuit is applied to the transformer terminals. An instant before the fault, the SG was delivering power to the system.

The system consists of a 2 pole, 150 MVA, 120 MW, 13.8 kV, 50 Hz SG connected to an infinite bus through a transformer and a transmission line, as shown in Figure 1.

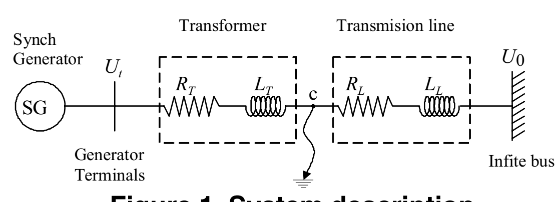

Figure 1. System description

The electric representation of the SG is done assuming a linear behavior of fluxes linkages and using two-axis equivalent circuits. This means that the three

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phase synchronous machine is transformed to an equivalent machine that has the rotor as the reference frame and concentrates its windings in two axes: the *d*-axis aligned with the field winding and the *q*-axis placed at 90°. This is accomplished using Adkins approach to Park's transformation [2]. The equivalent circuit parameters are inductances and resistances that represent the flux linkages and voltage drops of the two-axis machine windings which incorporate the electromagnetic phenomena taking place inside the generator. Figures 2 and 3 show the *d*- and *q*-axis equivalent circuits, where u_d, and u_q are the *d*- and *q*-axis winding voltages, u_f is the field voltage, R_{1d}, …, R_{nd}, R_{1q}, …, R_{nq} R_{1d}, …, L_{nd}, L_{1q}, …, L_{nq} are resistances and inductances of the damper windings of the corresponding axis, L_a and R_a are the resistance and inductance of the *d* and *q* stator windings, L_{fk1}, …, L_{fkn} are the mutual inductances of rotor windings [3] and $\omega\psi_{d,q}$ are the rotational voltages [2]. The order of the equivalent circuits depends on the number of damper circuits that are used to represent the solid-rotor eddy-current effect on each axis. The representation of the whole system is based on this two axis frame and referred to the infinite bus.

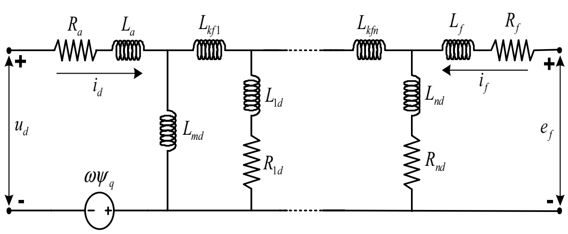

Figure 2. *d*-axis equivalent circuit

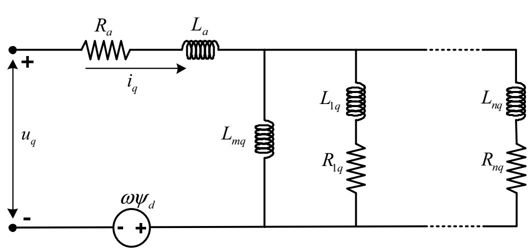

Figure 3. *q*-axis equivalent circuit

3. System state-space representation

There are different approaches to obtain a SS representation of SG [2,4,5,6]. The main difference is concerned with the selection of flux linkages or currents as the state variables for the electrical part of the system. In this work the state variables are currents (i), load angle (δ), angular velocity (ω) and field voltage (u_f).

The matrix form of the voltage equations for the generator is:

$$\mathbf{u} = \mathbf{R}\mathbf{i} + \frac{d}{dt}\mathbf{L}\,\mathbf{i} + \omega_m\,\mathbf{G}\,\mathbf{i} \qquad (1)$$

where ω_m is the angular velocity, **u** and **i** are the voltage and current vectors of the *d*- and *q*-axis windings:

$$\mathbf{i}^T = \begin{bmatrix} i_d & i_f & i_{1d} & \dots & i_{nd} & i_q & i_{1q} & \dots & i_{nq} \end{bmatrix}$$

$$\mathbf{u}^T = \begin{bmatrix} u_d & u_f & u_{1d} & \dots & u_{nd} & u_q & u_{1q} & \dots & u_{nq} \end{bmatrix}$$

where u_d, and u_q are the *d*- and *q*-axis stator winding voltages and u_f is the field voltage. The voltages of the damper windings u_{1d}, …, u_{nd}, u_{1q}, …, u_{nq} are zero. **R** and **L** are the resistance- and inductance-matrix:

$$\mathbf{R} = diag\begin{bmatrix} R_a & R_f & R_{1d} & \cdots & R_{nd} & R_q & R_{1q} & \cdots & R_{nq} \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} \mathbf{L_d} & \mathbf{\Theta}_{nd+2,nq+1} \\ \mathbf{\Theta}_{nq+1,nd+2} & \mathbf{L_q} \end{bmatrix}$$

where $\mathbf{\Theta}$ are zero matrices with their dimensions depending on the number of damper branches and

$$\mathbf{L_d} = \begin{bmatrix} L_{dd} & L_{df} & L_{d1d} & \cdots & L_{dnd} \\ L_{fd} & L_{ff} & L_{f1d} & \cdots & L_{fnd} \\ L_{1dd} & L_{1df} & L_{1d1d} & \cdots & L_{1dnd} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ L_{ndd} & L_{ndf} & L_{nd1d} & \cdots & L_{ndnd} \end{bmatrix}$$

$$\mathbf{L_q} = \begin{bmatrix} L_{qq} & L_{q1q} & \cdots & L_{qnq} \\ L_{1qq} & L_{1q1q} & \cdots & L_{1qnq} \\ \vdots & \vdots & \cdots & \vdots \\ L_{nqq} & L_{nq1q} & \cdots & L_{nqnq} \end{bmatrix}$$

the diagonal terms are the self inductances and the rest are mutual inductances. **G** is an array of the inductances of both axis, necessary to obtain the rotational voltages present in each stator winding:

$$\mathbf{G} = \begin{bmatrix} \mathbf{\Theta}_{1,nd+2} & \mathbf{G_q} \\ \mathbf{\Theta}_{nd+1,nd+2} & \mathbf{\Theta}_{nd+1,nq+1} \\ -\mathbf{G_d} & \mathbf{\Theta}_{1,nq+1} \\ \mathbf{\Theta}_{nq,nd+2} & \mathbf{\Theta}_{nq,nq+1} \end{bmatrix}$$

where

$$\mathbf{G_d} = \begin{bmatrix} L_{dd} & L_{df} & L_{d1d} & \cdots & L_{dnd} \end{bmatrix}$$

and

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$$\mathbf{G_q} = \begin{bmatrix} L_{qq} & L_{q1q} & \cdots & L_{qnq} \end{bmatrix}$$

The current SS model for the electric system is now established as:

$$\frac{d\mathbf{i}}{dt} = -(\mathbf{L}^{-1})(\mathbf{R} + \omega_m \mathbf{G})\mathbf{i} + (\mathbf{L}^{-1})\mathbf{u} \tag{2}$$

which corresponds to the form:

$$\frac{d\mathbf{i}}{dt} = \mathbf{Ai} + \mathbf{Bu} \tag{3}$$

where

$\mathbf{A} = -(\mathbf{L}^{-1})(\mathbf{R} + \omega_m \mathbf{G})$ and $\mathbf{B} = (\mathbf{L}^{-1})$

The mechanical part of the system can now be included using the swing equation:

$$\frac{d\omega_m}{dt} = \frac{\omega_0}{2H} T_m - \frac{\omega_0}{2H} T_e \tag{4}$$

where H is the inertia constant, T_m is the mechanical torque and T_e is the electrical torque:

$$T_e = \frac{\omega_0}{2}\left[\left(L_{dd} i_d + L_{df} i_f + L_{d1d} i_{1d}\right) i_q - \left(L_{qq} i_q + L_{q1q} i_{1q}\right) i_d\right] \tag{5}$$

The swing equation together with the change of load angle δ:

$$\frac{d\delta}{dt} = \omega_0 - \omega_m \tag{6}$$

form the state equations for the mechanical system.

The state equation for the automatic voltage regulator (AVR) corresponds to that originally installed in the SG:

$$\dot{u}_f = -\frac{u_f}{\mathrm{T}_r} + \frac{(u_r - u_t)K_r}{\mathrm{T}_r} \tag{7}$$

where u_f is the field voltage, u_r the reference voltage, u_t the voltage at the SG terminals, T_r a time constant and K_r the gain.

If the one-damper equivalent circuit is chosen to represent the SG and the mechanical system and the AVR are included, the state and input variables are:

$$\begin{aligned} \mathbf{x}^T &= \begin{bmatrix} i_d & i_f & i_{1d} & i_q & i_{1q} & \omega_m & \delta & u_f \end{bmatrix} \\ \mathbf{u}^T &= \begin{bmatrix} u_{dB} & 0 & u_{qB} & 0 & T_m & 1 & (u_r - u_t) \end{bmatrix} \end{aligned} \tag{8}$$

where the input variables u_{dB} and u_{qB} are the infinite bus d- and q-axis voltages, chosen as reference. They substitute the d and q components of the SG terminal voltages used in (2). These voltages are obtained as described in the next section.

The complete matrix representation of the equations of all state variables in (8) is given as:

$$\dot{\mathbf{x}} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & 0 & 0 & B_{12} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & 0 & 0 & B_{22} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & 0 & 0 & B_{32} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & 0 & 0 & B_{42} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & 0 & 0 & B_{52} \\ \frac{-\omega_0^2}{4H} L_{dd} i_q & \frac{-\omega_0^2}{4H} L_{df} i_q & \frac{-\omega_0^2}{4H} L_{d1d} i_q & \frac{\omega_0^2}{4H} L_{qq} i_d & \frac{\omega_0^2}{4H} L_{q1q} i_d & -\frac{D}{2H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{\mathrm{T}_r} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{1d} \\ i_q \\ i_{1q} \\ \omega_m \\ \delta \\ u_f \end{bmatrix} +$$

$$\begin{bmatrix} B_{11} & B_{13} & B_{14} & B_{15} & 0 & 0 & 0 \\ B_{21} & B_{23} & B_{24} & B_{25} & 0 & 0 & 0 \\ B_{31} & B_{33} & B_{34} & B_{35} & 0 & 0 & 0 \\ B_{41} & B_{43} & B_{44} & B_{45} & 0 & 0 & 0 \\ B_{51} & B_{53} & B_{54} & B_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\omega_0}{2H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{K_r}{\mathrm{T}_r} \end{bmatrix} \begin{bmatrix} u_{dB} \\ 0 \\ u_{qB} \\ 0 \\ T_m \\ 1 \\ u_r - u_t \end{bmatrix} \tag{9}$$

The values used for the parameters of the equivalent circuits were identified and validated in [7]. Parameters for up to five damper branches in the d-axis and four in the q-axis are available. The results reproduced here correspond to the one-damper branch case, as it is considered appropriate to describe the SS representation.

It is important to notice that apart from the addition of three state equations to the electric system the inclusion of the AVR equation produces important changes in the representation established in (2) where the field voltage u_f was an input variable and now in (9) is a state variable. This is an expected result of the inclusion of a closed loop system.

4. Initial conditions and terminal voltage

The initial conditions are obtained from measured data at the SG terminals: voltage, current and power factor. Since the chosen and logical reference for the analysis is the infinite bus and the SG is simulated with two-axis equivalent circuits, voltages at the infinite bus and in any other point of the system must be expressed in terms of the two-axis frame. Then the first step is to fix the position of the infinite bus voltage with respect to the SG two-axis frame, i.e. to find the load angle of the infinite bus voltage. This is performed using the phasor diagram of Figure 4, where the position of the voltage drop phasors due to the impedances of the SG-armature, transformer and transmission line are easily located (Figure 4). Then with the help of impedances angles and the power factor angle the magnitude of u_B and the load angle δ can be obtained. Once δ is known the steady state values of infinite bus voltage components u_{dB} and u_{qB} and line current components i_d, and i_q are found. The initial value of the field current can be derived from the steady state expression of u_{qB}:

$$u_{qB} = (R_a + R_T + R_L)i_q - \omega_0 L_{md} i_f - \omega_0 (L_{md} + L_a + L_T + L_L) i_d$$

and the field voltage can then be obtained:

$$u_f = R_f i_f$$

Finally the steady state value of mechanical torque:

$$T_m = \frac{\omega_0}{2}\left[\left(L_{dd} i_d + L_{df} i_f\right) i_q - \left(L_{qq} i_q\right) i_d\right]$$

and the steady state value of currents in all damper windings are zero. This procedure is equivalent to making $\dot{\mathbf{x}} = 0$.

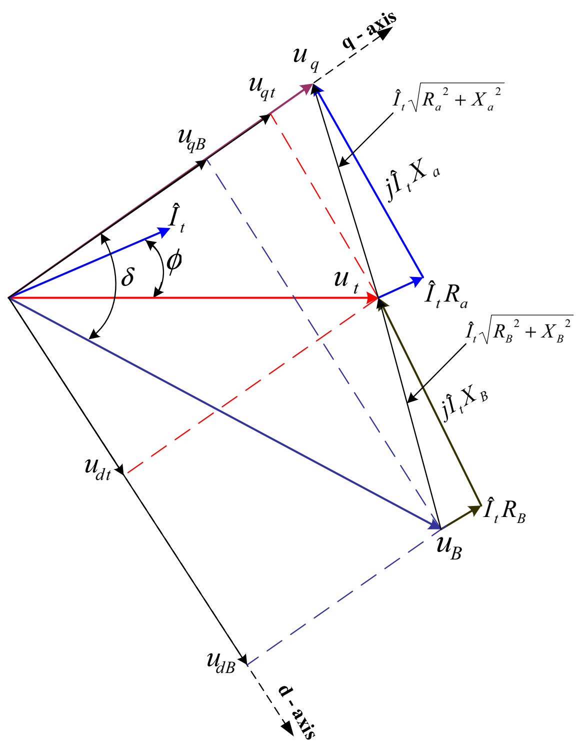

Figure 4. Phasor diagram to obtain infinite-bus voltages according to the on load conditions of the generator

After the SS equations have been solved the SG terminal voltage can be calculated:

$$u_t = \frac{\sqrt{u_{dt}^2 + u_{qt}^2}}{\sqrt{2}}$$

where the d- and q-axis stator voltages u_{dt} and u_{qt} are calculated differently depending if it is the fault period:

$$u_{dtFault} = -R_T i_d - L_T \dot{i}_d - L_T i_q (\omega_m - \dot{\delta})$$
$$u_{qtFault} = -R_T i_q - L_T \dot{i}_q + L_T i_d (\omega_m - \dot{\delta})$$

where R_T and L_T are the transformer resistance and inductance, or the postfault period:

$$u_{dtPostfault} = \sqrt{2}\, u_{dB} - (R_T + R_L) i_d - (L_T + L_L)\dot{i}_d - (L_T + L_L) i_q (\omega_m - \dot{\delta})$$
$$u_{qtPostfault} = \sqrt{2}\, u_{qB} - (R_T + R_L) i_q - (L_T + L_L)\dot{i}_q + (L_T + L_L) i_d (\omega_m - \dot{\delta})$$

where R_L and L_L are the line resistance and inductance.

5. Software and results

The software code was written for the MATLAB environment [8]. It uses the *ode45* Runge Kuta solver and contemplates the possibility of including any number of damper branches in the equivalent circuits. The results given here correspond to the one-damper branch equivalent circuits. The curves obtained are shown in Figures 5 to 11. These figures include the measured data (filtered and recorded for 4 seconds) of the generator response (marked as USK) used as a reference [9], as well as the results obtained with a FE model [10]. The three phase short circuit is applied on time zero and it lasts 0.14 seconds.

In analyzing the differences found in the graphics one should bear in mind that although the SS model provides a strong coupling between systems, the representation of the AVR was proposed from tests and therefore is approximate [9]. It is also important to know that the SS representation does not include the turbine-governor effect. The FE model uses separate subroutines to include the effect of the AVR and governor [10].

It is worth mentioning some comments about the graphics. The frequency and amplitude of oscillations of measured data in Figures 5 to 9 are always smaller than the calculated data and hence the best fitness of curves is found on the first period of the 1.25 Hz oscillations, with the SS model being the one that better captures the first oscillation of the load angle (Figure 5). The differences in frequency and amplitude could be due to friction which is neglected in the FEM and SS models, as well as the damping of the current trajectories in the rotor that are represented with only one damping branch in the models.

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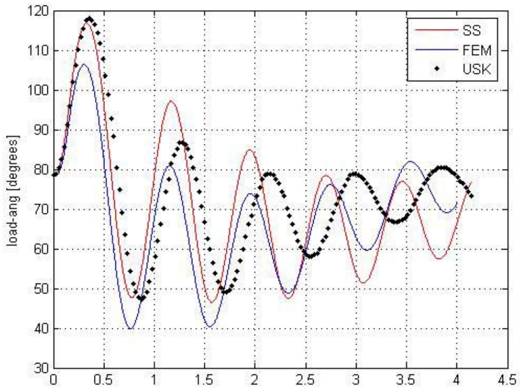

Figure 5. Load angle (SS: state-space, FEM: FE model, USK: measured)

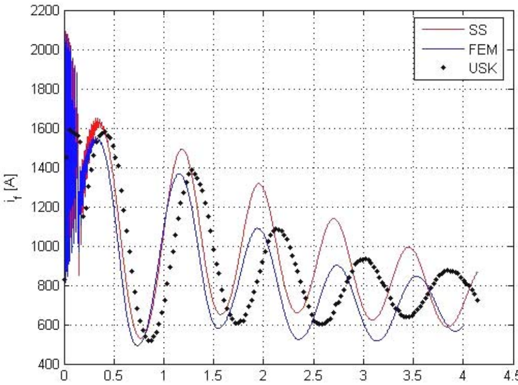

Figure 8. Field current (SS: state-space, FEM: FE model, USK: measured)

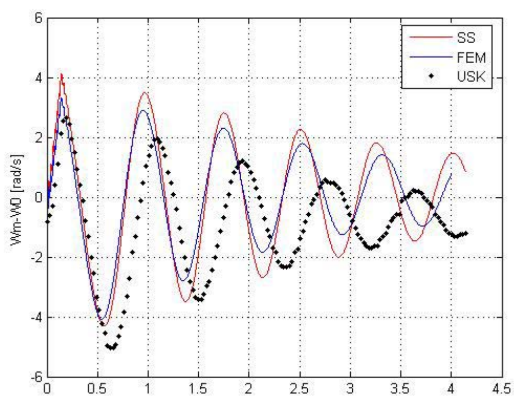

Figure 6. Speed deviation (SS: state-space, FEM: FE model, USK: measured)

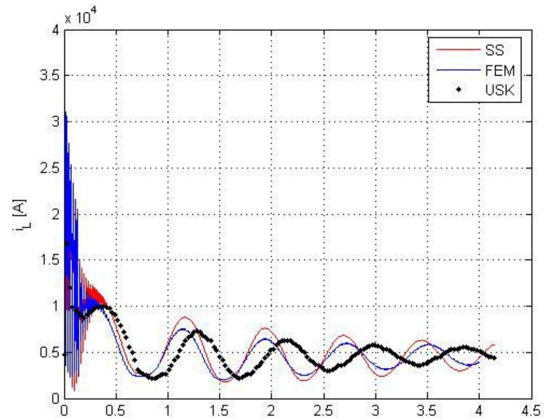

Figure 9. Line current (SS: state-space, FEM: FE model, USK: measured)

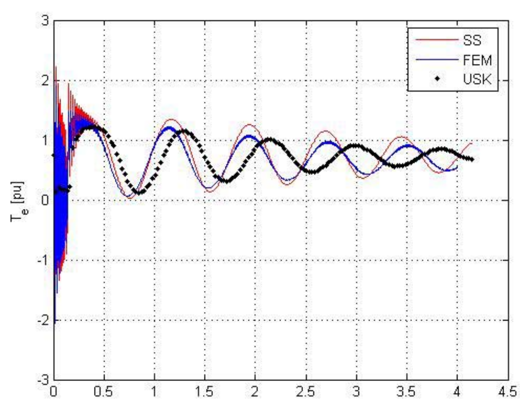

Figure 7. Electromagnetic Torque (SS: state-space, FEM: FE model, USK: measured)

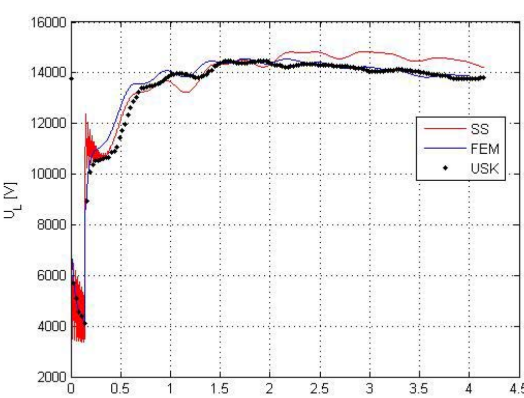

Figure 10. Line voltage (SS: state-space, FEM: FE model, USK: measured)

It is interesting to see that in the speed deviation curves (Figure 6) both models capture the first swing effect, where the fault abrupt change is opposed by the flux linkages and in fact decelerates the generator. After the short speed decrement, the rotor accelerates until the fault is tripped. The FE model and the SS representation have important similarities that at the same time are close to the measured results which give confidence in both models.

Electromagnetic torque, line current and line voltage (Figures 7, 9 and 10) show a good fitness of both models. On the other hand the differences found in the speed deviation and in the load angle (Figures 5 and 6) could be caused by the fact that the turbine governor effect is not included.

6. Conclusions

A SS representation of a SG was established. The state-variable equations include the two-axis equivalent circuits, the swing equation and the effect of voltage regulator. The system was solved using *ode45* solver of MATLAB. The usefulness of the SS model is illustrated with the results of the one-damper branch equivalent circuits.

The comparison made of the SS representation with measured data and with the FE model gives satisfactory results. The intention of validating a SS model was accomplished and this established representation can now be used to continue with the analysis of the generator stability using other control approaches like the one used in [11].

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