Power Factor

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Casa abierta al tiempo
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Content

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Recommended Bibliography

The recommended bibliography is the following:

- J. J. Grainger and W. D. Stevenson, *Power System Analysis*, McGraw-Hill Education, 2003.
- Charles Alexander and Matthew Sadiku, *Fundamentals of Electric Circuits*, Fifth Edition, McGraw-Hill Higher Education, 2012.
- Mahmood Nahvi, Joseph Edminister and William Travis Smith, *Schaum's Easy Outline of Electric Circuits*, McGraw Hill Professional, 2004.

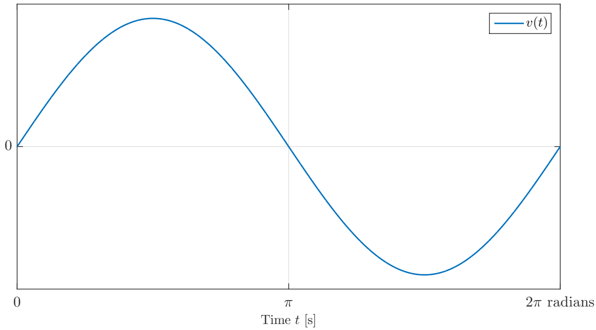
Instantaneous Power

The instantaneous power *p(t)*, in watts, is the demanded power at any time by a load in an installation, product of the instantaneous voltage at its terminals *v(t)* and the instantaneous current flowing *i(t)*:

$$p(t) = v(t)i(t)$$

Instantaneous Power

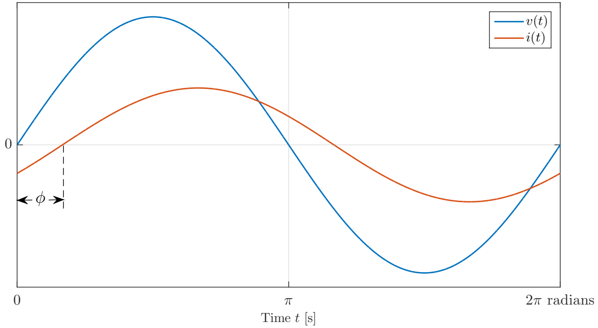
- Voltage (V): Voltage or potential difference, is the required power to move a unit load through an element, measured in volts [V].



For a frequency of 50Hz the time for a cycle is 1/50 s.

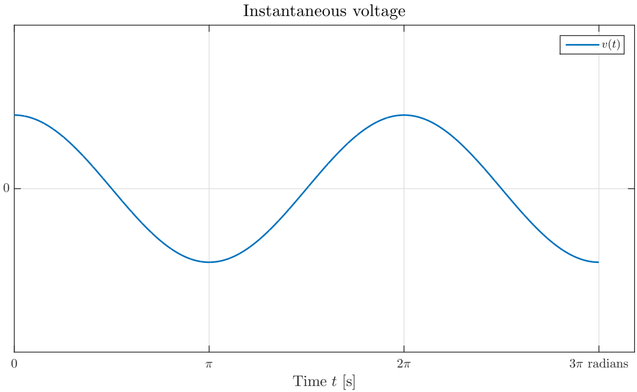
Instantaneous Power

- Voltage (V): Voltage or potential difference, is the energy required to move a unit load through an element, measured in volts [V].
- Current (I): Electric Current is the rate of change of the charge with respect to the time, measured in amperes [A].



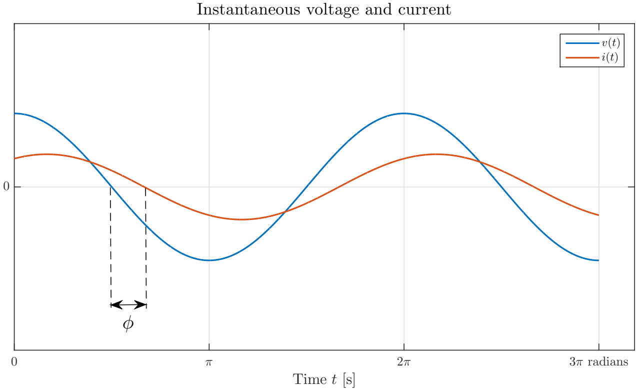
For a frequency of 50Hz the time for a cycle is 1/50 s.

Instantaneous Power



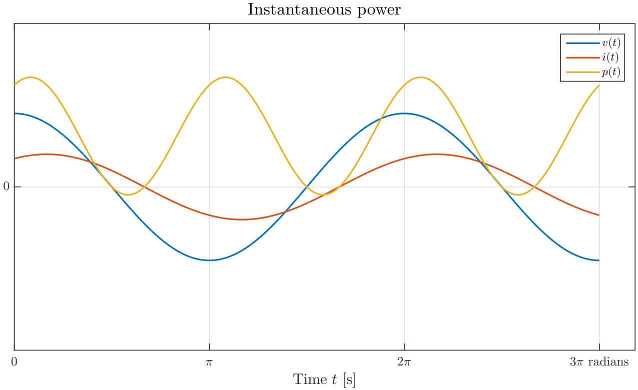
$$v(t) = V_m \cos(\omega t)$$

Instantaneous Power



$$v(t) = V_m \cos(\omega t)$$
$$i(t) = I_m \cos(\omega t - \phi)$$

Instantaneous Power



$$p(t) = v(t)i(t)$$

Instantaneous Power

For sinusoidal voltage and current:

$$v(t) = V_m \cos(\omega t)$$
$$i(t) = I_m \cos(\omega t - \phi)$$

the instantaneous power *p(t)* can be written:

$$p(t) = V_m I_m \cos(\omega t) \cos(\omega t - \phi)$$

where V_m and I_m are the amplitudes or maximum values, ϕ is the lagging angle or angle of delay of the current i with respect to the voltage v and ω=2πf is the synchronous speed that is measured in rad/s.

Instantaneous Power

The instantaneous power *p(t)* can be written:

$$p(t) = V_m I_m \cos(\omega t) \cos(\omega t - \phi)$$

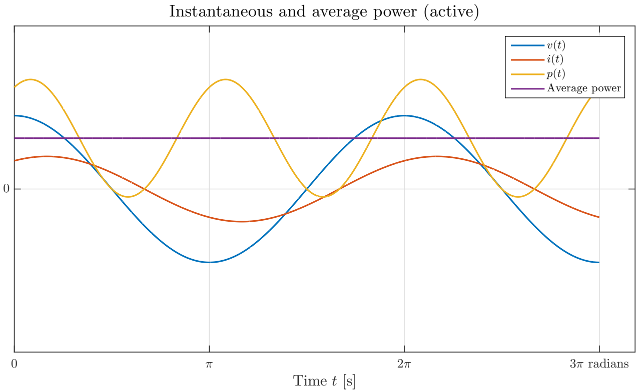
using the trigonometric identity:

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

the instantaneous power *p(t)* expression becomes:

$$p(t) = \frac{1}{2} V_m I_m \cos(\phi) + \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$$

Average Power (active)



$$P = \frac{1}{T}\int_0^T p(t)dt$$

Average Power (active)

The active power P, in watts, is the average of the instantaneous power $p(t)$ along a period T.

$$P = \frac{1}{T}\int_0^T p(t)dt$$

thus:

$$P = \frac{1}{T}\int_0^T \left[\frac{1}{2}V_m I_m \cos(\phi) + \frac{1}{2}V_m I_m \cos(2\omega t - \phi)\right] dt$$

the expression for active power P becomes:

$$P = \frac{1}{2}V_m I_m \cos\phi$$

RMS

The Root Mean Square (RMS) or effective value of an AC current corresponds to the value of a DC current that would be needed to dissipate the same amount of heat from a resistance. For any periodic function $x(t)$, the RMS value is given by:

$$X_{RMS} = \sqrt{\frac{1}{T}\int_0^T x^2 dt}$$

Please determine the RMS value for the AC cosine wave voltage $v(t) = V_m \cos(\omega t)$.

It is important to remember the following trigonometric identity:

$$\cos^2 x = \tfrac{1}{2} + \tfrac{\cos 2x}{2}$$

RMS

In the case of a sinusoidal voltage $v(t) = V_m \cos(\omega t)$, its RMS value is:

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

similarly in the case of $i(t) = I_m \cos(\omega t)$:

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

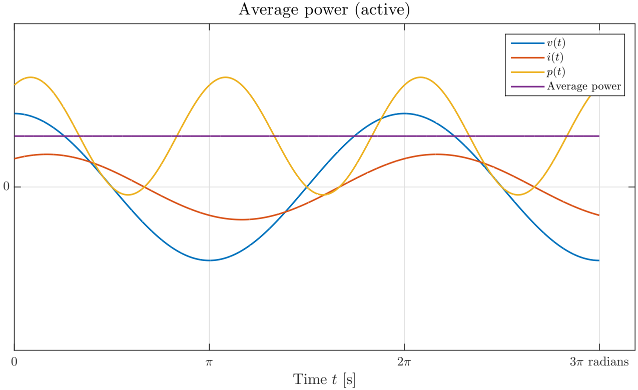
where V_m and I_m are the amplitudes or maximum values and $\omega = 2\pi f$ is the synchronous speed.

Active Power

Effective values (RMS) are now used.

$$P = \frac{1}{2} V_m I_m \cos(\phi) = V_{RMS} I_{RMS} \cos\phi$$

Active Power (average)



$$P = V_{RMS} I_{RMS} \cos\phi$$

Phasors

A phasor is a complex number that represents the amplitude and phase of a sinusoid. The use of phasors facilitates the analysis of AC electrical networks operating in steady state, allowing to represent sinusoidal time-varying functions as a complex number in the frequency domain. Phasors simplify calculations since only algebra of complex numbers is needed.

Phasors can only be used under the following assumptions:

- The circuit is linear.
- The circuit is excited by sinusoidal sources.
- Steady state.

Conversion Time-Phasor

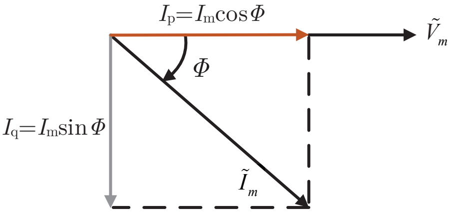
To obtain the phasor of a sinusoid, the function should be written as a cosine rather than a sine. The transformation is summarized as follows:

$$i(t) = I_m \cos(\omega t + \phi) \;\Leftrightarrow\; \tilde{I} = I_m \angle \phi$$

$$i(t) = I_m \sin(\omega t + \phi) = I_m \cos[\omega t + (\phi - 90)] \;\Leftrightarrow\; \tilde{I} = I_m \angle \phi - 90$$

Phasor Diagrams of Current

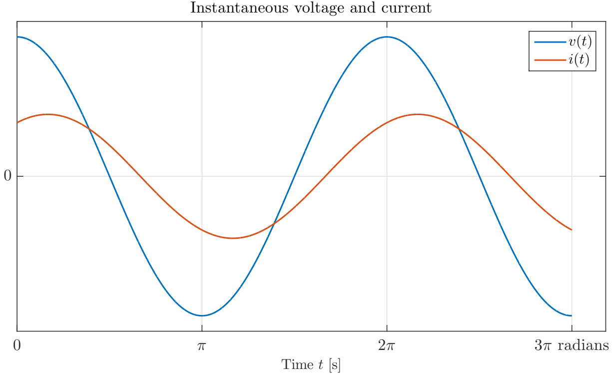
In phasor form I_m can be split into: I_p in phase with the voltage and I_q in quadrature with the voltage.



Phasor Diagrams of Current

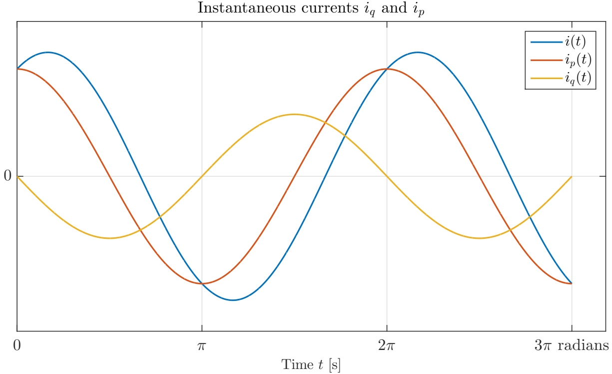
Please express the following sinusoids as phasors: $v(t) = -10\cos(2t + 40°)$ and $i(t) = 2\sin(10t + 15°)$.

Phasor Diagrams of Current



$$v(t) = V_m \cos(\omega t)$$
$$i(t) = I_m \cos(\omega t - \phi)$$

Phasor Diagrams of Current



$$i_p = I_m \cos\phi \cos\omega t = I_p \cos\omega t$$
$$i_q = -I_m \sin\phi \sin\omega t = -I_q \sin\omega t$$

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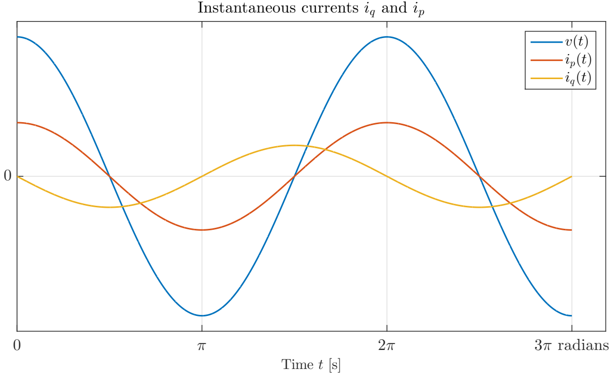
Phasor Diagrams of Current



$$i_p = I_m \cos\phi \cos\omega t = I_p \cos\omega t$$
$$i_q = -I_m \sin\phi \sin\omega t = -I_q \sin\omega t$$

Currents Definition

The currents in the time domain:

$$i_p = I_m \cos\phi \cos\omega t = I_p \cos\omega t$$

The active power is the product $V_m I_p$.

$$i_q = -I_m \sin\phi \sin\omega t = -I_q \sin\omega t$$

Remains to consider the product $V_m I_q$.

Instantaneous Power

Now the instantaneous power can be written as:

$$p(t) = v(t)[i_p(t) + i_q(t)]$$

$$p(t) = V_m I_p \cos\omega t \cos\omega t - V_m I_q \cos\omega t \sin\omega t$$

$$p(t) = \frac{V_m I_p}{2}(1 + \cos 2\omega t) - \frac{V_m I_q}{2}\sin 2\omega t$$

It is important to remember the following trigonometric identities:

$$\cos^2 x = \frac{1}{2} + \frac{\cos 2x}{2}$$
$$\sin x \cos x = \frac{\sin 2x}{2}$$

Active Power P

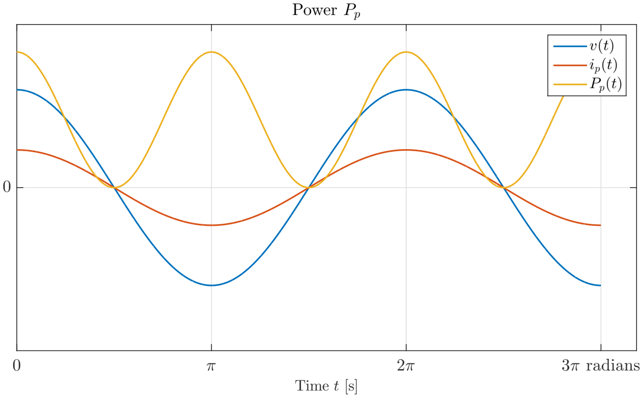
Now we can conveniently define p_p(t) as:

$$p_p(t) = \frac{V_m I_p}{2}(1 + \cos 2\omega t)$$

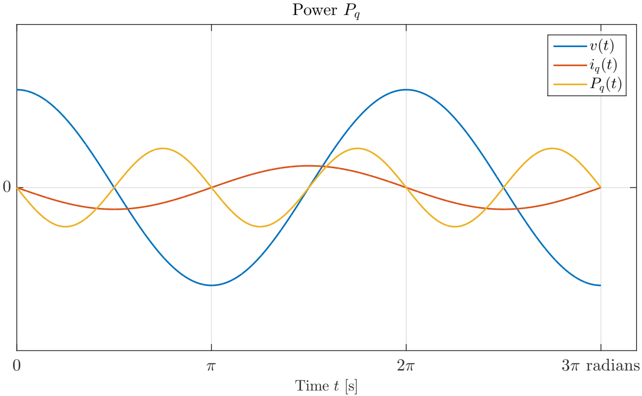
The average power of p_p(t) is:

$$P = \frac{1}{2} V_m I_m \cos(\phi) = V_{RMS} I_{RMS} \cos \phi$$

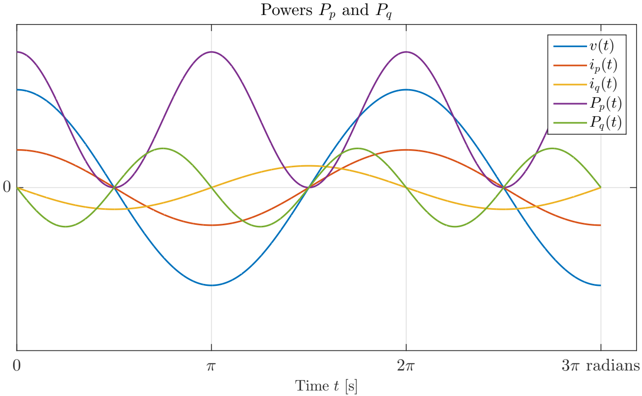
P_p and P_q Powers



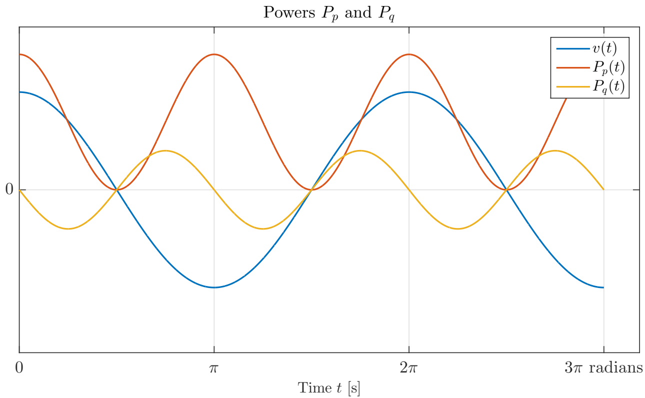
P_p and P_q Powers



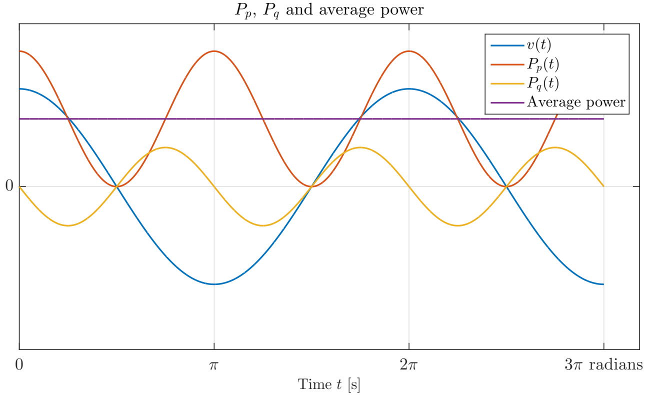
P_p and P_q Powers



P_p and P_q Powers



P_p and P_q Powers



$$P = V_{RMS} I_{RMS} \cos \phi$$

Reactive Power Q

Now we can define p_q(t) as:

$$p_q(t) = -\frac{V_m I_q}{2} \sin 2\omega t$$

The average component p_q(t) is zero, but the wave has a maximum value given by:

$$p_{q,\max} = \left| -\frac{V_m I_q}{2} \right| = |V_{RMS} I_{RMS} \sin \phi|$$

Reactive Power Q

Reactive power Q, measured in VAR (reactive volt-amperes), is a measure of the energy exchange between the power supply and the reactive part of the load (energy storage elements).

$$Q = \frac{1}{2} V_m I_m \sin(\phi) = V_{RMS} I_{RMS} \sin \phi$$

Apparent Power

The apparent power S, measured in VA (volt-amperes), is so called because *apparently* the power should be the current-voltage product as in the CD circuits.

$$S = \frac{1}{2} V_m I_m = V_{RMS} I_{RMS}$$

Apparent power is used to calculate the size of the conductors and other elements of an electrical installation, as the need to have the capacity for the current that will satisfy the demand of active and reactive power.

Complex Power

The complex power **S**, in VA, is the product of the phasor voltage and current phasor conjugate (it is the same complex number but with the sign of the imaginary part changed). Its real part represents the active power P and its imaginary part the reactive power Q. If the current phasor conjugate is not used, the imaginary part of **S** does not represent the reactive power Q. The use of complex power allows for the powers S, P and Q to be obtained using voltage and current phasors.

$$
\begin{aligned}
\mathbf{S} &= \frac{1}{2}\tilde{V}_m \tilde{I}_m^{\star} = \tilde{V}_{RMS} \tilde{I}_{RMS}^{\star} \\
S = |\mathbf{S}| = V_{RMS} I_{RMS} &= \sqrt{P^2 + Q^2} \\
P &= \mathrm{Re}(\mathbf{S}) \\
Q &= \mathrm{Im}(\mathbf{S})
\end{aligned}
$$

Power Factor

The power factor (pf) is the ratio of the active power consumed by the load and the apparent power of the load. It can be defined as the cosine of the phase difference between voltage and current.

$$pf = \frac{P}{S} = \cos\phi$$

Power Expressions

$$S = \frac{1}{2}V_m I_m = V_{RMS} I_{RMS}$$

$$P = \frac{1}{2}V_m I_m \cos(\phi) = V_{RMS} I_{RMS} \cos(\phi) = S \cos\phi$$

$$Q = \frac{1}{2}V_m I_m \sin(\phi) = V_{RMS} I_{RMS} \sin(\phi) = S \sin\phi$$

$$pf = \cos(\phi) = \frac{P}{S}$$

All supplying companies have a surcharge formula for low power factor.

Power Expressions

$$S = \frac{1}{2}V_m I_m = V_{RMS} I_{RMS}$$

$$P = \frac{1}{2}V_m I_m \cos(\phi) = V_{RMS} I_{RMS} \cos(\phi) = S \cos\phi$$

$$Q = \frac{1}{2}V_m I_m \sin(\phi) = V_{RMS} I_{RMS} \sin(\phi) = S \sin\phi$$

$$pf = \cos(\phi) = \frac{P}{S}$$

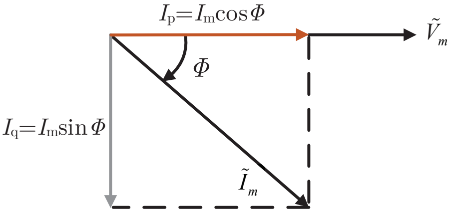
All supplying companies have a surcharge formula for low power factor.

Please, calculate the S, P, Q y pf for a load that demands a current of $i(t) = 4\cos(\omega t + 10)$ when it is connected to a power source whose voltage is $v(t) = 120\cos(\omega t - 20)$.

Power Expressions

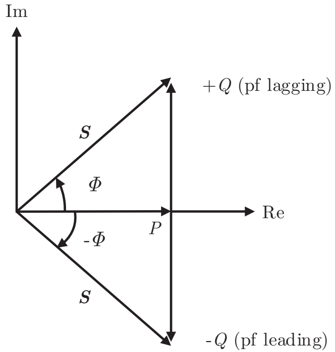
$$\phi = -30^\circ$$
$$pf = \cos\phi = \cos(-30) = 0.866(leading)$$
$$P = S\cos\phi = 207.84 \quad \text{W}$$
$$Q = S\sin\phi = -120 \quad \text{VAR}$$

It is important to note that we have a *power factor leading* because of ϕ is negative.



Power Triangle

It is common to represent **S**, P and Q with a triangle called *Power Triangle*.



Phasor Diagrams

Before continuing, it is necessary to introduce the following definitions:

- Resistance (R): The resistance of an element is its ability to resist the flow of electric current, measured in ohms [Ω].
- Inductance (L): The inductance is the property by which an inductor has opposition to the change of the current flowing through it, measured in henrys [H]. An inductance causes that the current wave lags the voltage wave.
- Capacitance (C): The capacitance is the proportion of the charge on a plate of a capacitor and the potential difference between the two plates, measured in farad [F]. A capacitance causes that the current wave leads the voltage wave.
- Reactance (X): The reactance is the opposition of inductors and capacitors to the flow of an alternate current, measured in ohms [Ω].

Phasor Diagrams

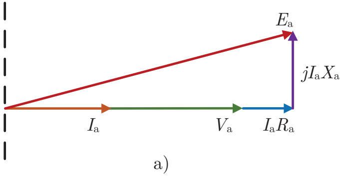
$\tilde{E}_a = \tilde{V}_a + \tilde{I}_a(R_a + jX_a)$

V_a: terminal voltage

E_a: generated voltage

I_a: armature current

X_a: leakage reactance



Phasor diagrams for a power factor: a) unit.

Phasor Diagrams

$$\tilde{E}_a = \tilde{V}_a + \tilde{I}_a(R_a + jX_a)$$

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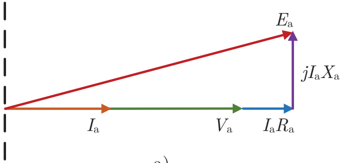
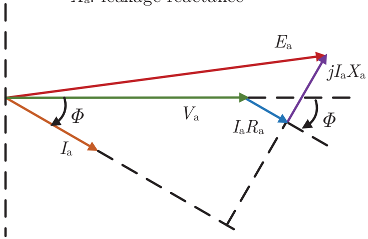


a)



b)

Phasor diagrams for a power factor: a) unit and b) lagging.

Phasor Diagrams

$$\tilde{E}_a = \tilde{V}_a + \tilde{I}_a(R_a + jX_a)$$

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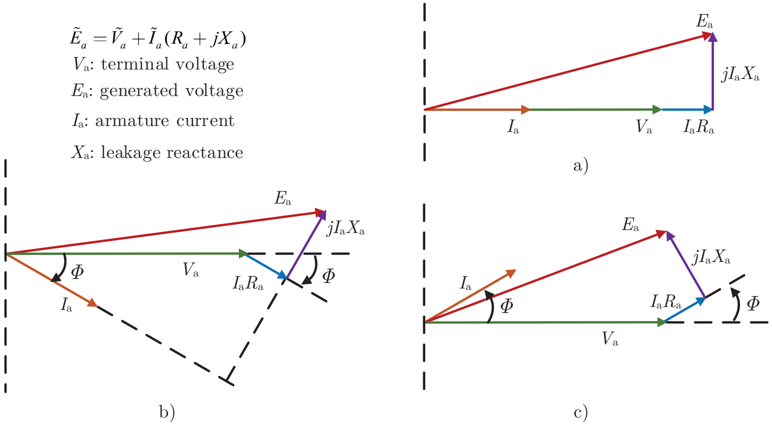
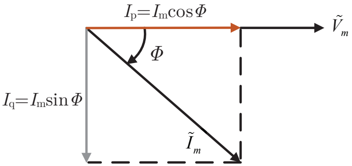
X_a: leakage reactance



Phasor diagrams for a power factor: a) unit, b) lagging and c) leading.

Correction of Power Factor

Before continuing, it is necessary to remember that in phasor form I_m can be split into: I_p in phase with the voltage and I_q in quadrature with the voltage.



So, using RMS values we can defined a reactance as follows:

$$X = \frac{V_m}{I_q} = \frac{V_m}{I_m \sin\phi} = \frac{V_m^2}{V_m I_m \sin\phi} = \frac{V_m^2}{Q}$$

Note: V_m, I_m and I_q are RMS values.

Correction of Power Factor

So, we can defined a reactance as follows:

$$X = \frac{V_m}{I_q} = \frac{V_m}{I_m \sin\phi} = \frac{V_m^2}{V_m I_m \sin\phi} = \frac{V_m^2}{Q}$$

Also, capacitive reactance (X_C) and inductive reactance (X_L) are defined as follows:

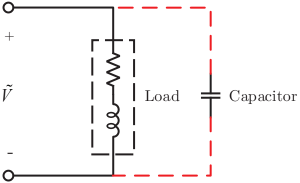
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad X_L = \omega L = 2\pi f L$$

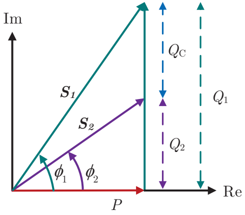
Therefore, the capacitance and inductance can be computed with the following equation:

$$C = \frac{Q}{\omega V_m^2} \qquad L = \frac{V_m^2}{\omega Q}$$

Correction of Power Factor

The correction of power factor is the process of increasing the *pf* without altering the voltage or current, and therefore the active power of the original charge.





$$C = \frac{Q_C}{\omega V_{RMS}^2} = \frac{Q_1 - Q_2}{\omega V_{RMS}^2}$$

Variables with the subscript 1 and 2 correspond to the original charge and the corrected load, respectively. Q_C is the reactive power supply needed and C is the required value of capacitor in parallel.

Correction of Power Factor

Please, make the correction of power factor to 0.95(-) for a load that consume 4 kW with pf=0.80(-) when it is connected to a power source whose frequency and voltage are 60 Hz and 120 V (RMS), respectively.

Power Factor in Electrical Installations

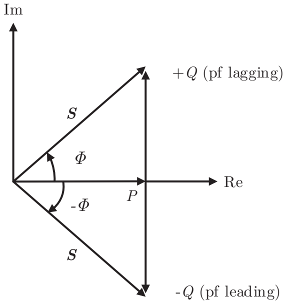
Equipment fed with electricity in buildings usually have motors or transformers or other type of equipment that operate with magnetic fluxes. This type of equipment requires active and reactive power for its operation and they produce a current delay which is define as lagging *pf*. This has a direct impact on voltage regulation and on the losses in the conductors that transport electricity to this type of equipment.

As seen before the power factor is defined as:

$$pf = \cos\phi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$

Power Factor in Electrical Installations

And the phasor diagram of active and reactive power is shown below:

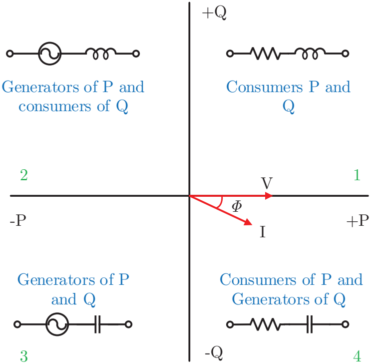


The pf can also be expressed in terms of the current components:

$$pf = \cos\phi = \frac{I_p}{I} = \frac{I_p}{\sqrt{I_p^2 + I_q^2}}$$

Power Factor in Electrical Installations

As illustrated in the figure, a resistive-inductive element consumes P and Q, while a resistive-capacitive element consumes P but generates Q. The quadrants “2” and “3” require active power generation and correspond to active elements, such as synchronous alternators. The active and reactive power can be found by projecting V along the axes P and Q as follows: P= VI$\cos\phi$ y Q= VI$\sin\phi$.



Power Factor Correction

The load of electrical installations is mainly resistive and inductive, so they normally have a lagging *pf*. This can mean a surcharge from the supplier but also represents losses.

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There are several options to correct the *pf*, but the more common is to install capacitor banks that will supply the reactive kVA needed to have a *pf* close to unity. It is important to mention that capacitor banks should not be installed in circuits that have waves with a high harmonic content.

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In industrial installations the *pf* can be maintained near unity if synchronous motors are used instead of induction motors. Synchronous motors have a self-excitation source that provides all the reactive power needed for their operation and can also provide more reactive power and compensate other equipment demand of reactive power.

Capacitor Bank Determination and Location

The amount of kVAR (reactive kVA) to improve the *pf* can be determined if the demand of reactive power required by the installed equipment is known. If the information is not available, the *pf* can be determined when the installation is completed and can start operation. The measurement of the first month consumption of kWh and kVARh are then used to calculate *pf*:

$$pf = \cos\left(\tan^{-1}\frac{kWh}{kVARh}\right)$$

Capacitor Bank Determination and Location

Then the initial condition is:

$$S_1 = \frac{P}{\cos\phi_1} \text{ and } Q_1 = \sqrt{S_1^2 - P^2}$$

and the desired condition is:

$$S_2 = \frac{P}{\cos\phi_2} \text{ and } Q_2 = \sqrt{S_2^2 - P^2}$$

The three-phase capacitor bank capacity is then:

$$Q_{bank} = \sqrt{3} V_{bank} I_{bank} = Q_1 - Q_2$$

This will be the capacitor bank for the whole installation but to reduce losses it is better to install capacitors close to each motor or equipment that requires reactive power. This will reduce the circulation of current in conductors feeding those parts of the electrical installation and will avoid the generation of losses.

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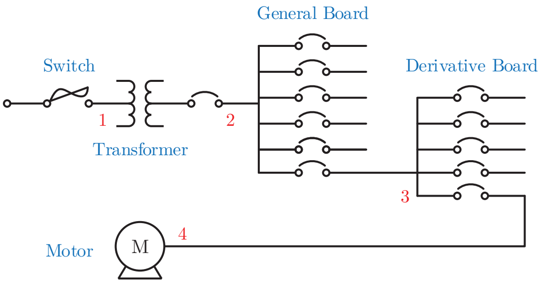
The following single line diagram gives an idea of the places where the capacitors can be located, to avoid losses the capacitors should be next to the load demanding reactive power, position “4” in diagram. The position “1” will have the maximum losses as it will need a transformer that considers the current demanded by active and reactive power. Locating capacitors next to the equipment will have an impact in the initial cost of the installation.



Capacitor Bank Determination and Location

It is very important to have ways to measure power factor and to correct it so it gets closer to the 1 pf.